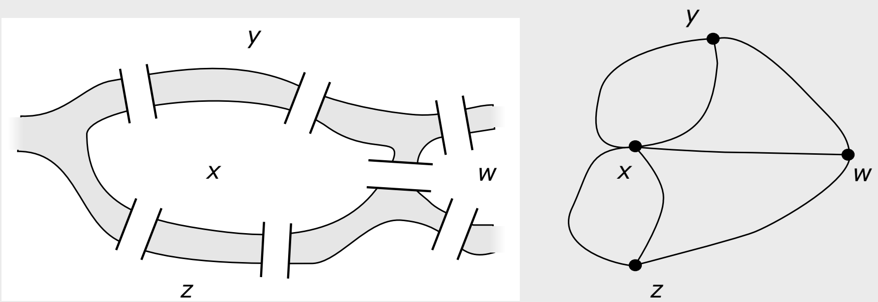
Königsberg bridge problem *The Königsberg bridge problem is the question of whether it is possible to walk a circuit that crosses each of seven bridges (shown below left) in the city of Königsberg once and only once. You might try to find such a route yourself. However, don't try for too long as in 1786 the mathematician Leonard Euler succeeded in proving that the task is impossible. Euler's proof is based on a model of the topological relationships between the Königsberg bridges (shown below right). The nodes, labeled* w, x, y, *and* z, *are abstractions of the regions of dry land. The edges between nodes are abstractions of the bridges connecting regions of dry land. Euler noted that apart from the start and end nodes, the path through a node must come in along one edge and out along another edge. So, if the problem is to be solvable then the number of edges incident with each intermediate node must be even. However, in Königsberg none of the nodes is incident with an even number of edges. Thus, Euler proved that it was impossible to cross each bridge just once.*



number of edges with which it is incident, and so the degree of all the nodes in G is three. A *path* between two nodes is a connected sequence of edges between the nodes, and is usually denoted by the nodes that it passes through. Examples of paths between nodes a and d in G are afd, acd, abd, and $abecafd$. A *connected graph* is such that there exists a path between any two of its nodes: G is connected.

path

connected graph

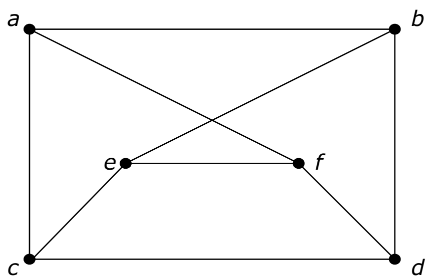


Figure 3.40: A graph G with six nodes and nine edges

Two graphs may show exactly the same connectivity relationships, and such graphs are said to be *isomorphic*. Sometimes isomorphism can be hard to detect, because a graph can disguise itself quite well. Thus, graph H in Figure 3.41 is isomorphic to graph G in Figure 3.40, since it has precisely the same nodes and edges. In this case, we have made the case clearer by labeling the nodes to show the isomorphism.